# Losses Approximation for Soft Magnetic Composites Based on a Homogenized Equivalent Conductivity

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Soft magnetic composites (SMC) are a promising alternative to laminated steel in many Electrical Engineering applications. This is largely owing to their low level of eddy current losses. The electromagnetic behavior of SMC in electromagnetic devices cannot be easily predicted using standard numerical techniques such as the finite element method, mostly due to the computational cost required to model the material microstructure. Another difficulty lies in the high property contrast between the matrix and the inclusions. In this paper we propose a homogenization strategy to define the equivalent electromagnetic properties of SMC. These equivalent properties can be used to calculate eddy current losses or introduced into structural analysis tools to design electromagnetic devices.

Index Terms—Magnetic losses, composite materials, homogenization, electric conductivity.

## I. INTRODUCTION

THE MICROSTRUCTURE of Soft Magnetic Composites (SMC), consisting of ferromagnetic inclusions embedded in a dielectric polymer matrix, explains the low level of eddy current (EC) losses observed in these materials when they are subjected to an electromagnetic loading. This is why they are considered as a promising alternative to laminated steel, for instance in motors [1]-[2].

In order to design electrical machines using SMC as magnetic material, it is critical to have an accurate estimate of losses. Finite Element Method (FEM) provides a full-field approach to compute EC losses. In the case of SMC, the microstructure has to be finely meshed and the brute FEM approach can bring significant numerical burden and instabilities. Various strategies derived from standard FEM have been proposed [3]-[4] to reduce to a certain extent the computational time and resources while retaining accuracy. However, the application to SMC can remain highly unstable, notably due to the high property contrast between the matrix and the inclusions.

Homogenization techniques can be used to determine the effective properties of composite materials. The corresponding homogeneous equivalent properties can then be used in standard structural analysis tools. Such techniques have been applied to the determination of the effective magnetic permeability of ferromagnetic polycrystals[5]-[6] or to the determination of the effective permittivity of composites for shielding applications [7]-[9]. In this paper, an analytical homogenization strategy is proposed to estimate the EC losses in SMC under standard operating conditions for electrical machines.

In a first part, homogenization is briefly presented and the problems to homogenize SMC are explained. In a second part, a strategy to overcome these problems is detailed, defining the equivalent conductivity from EC losses equivalence.

#### II. HOMOGENIZATION OF LOSSES IN SMC

Homogenization can be a useful tool when the numerical study of a device involves different scales, which would lead to unmanageable numerical system. The principle of homogenization is to replace heterogeneous materials with homogeneous media exhibiting the same macroscopic behavior. For SMC, the parameters of interest are the effective permeability and the level of losses.

Since this study is limited to low frequency, effective permeability can be deduced from classical homogenization approaches (Wiener or Maxwell-Garnett estimates). However, trying to homogenize the effective conductivity in the same manner will lead to a wrong loss estimate because of a very different current distribution in heterogeneous and homogeneous cases. Then, we propose to define an equivalent conductivity instead, which provides a proper loss estimate.

#### A. EC losses in SMC

The EC losses *U* are defined as the Joule losses dissipated per unit volume during a wave period:

$$U = \frac{\langle \mathbf{E}^* \overline{\sigma} \mathbf{E} \rangle}{2f} \tag{1}$$

where f is the frequency of the electric field **E** within the considered domain and  $\overline{\sigma}$  is the electric conductivity tensor. The operator  $\langle \cdot \rangle$  denotes a volume average over the domain and the superscript \* symbol refers to conjugate transpose.

EC losses in SMC are intricately connected to the material microstructure and structure (geometry). A model of losses in SMC has then to incorporate a description of the microstructure. The microstructure is approximated here as a periodic pattern of ferromagnetic inclusions embedded in a dielectric domain.

A time-harmonic magnetic flux is imposed perpendicularly to the domain (*z* direction). EC can be separated into a part contained inside the ferromagnetic inclusions (in-grain current linked to the microstructure) and another part flowing from grain to grain through the matrix (global current linked to both microstructure and structure). When the conductivity contrast ( $c_{\sigma} = \sigma_2/\sigma_1$ ) between grains and matrix is high and the exciting frequency low, the second contribution is negligible. Under such conditions, the interaction between grains can be neglected for the calculation of EC losses. The global losses are then limited to the sum of the losses in all independent grains. For the microstructure considered here, EC losses in each elementary cell can be calculated analytically.

### B. Problematics with homogenization of SMC

Contrary to EC in the heterogeneous SMC, the induced EC appearing in a homogeneous material is only a global current. The distribution of this global current is linked to the structure (geometry) of the material.

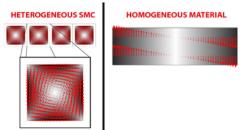


Fig. 1. EC distribution in heterogeneous and homogeneous material for a plate-shaped structure.

The use of homogenization to determine losses is then very difficult to perform since the EC distributions are totally different in the heterogeneous and homogeneous cases and cannot be readily deduced one from the other.

## III. DEFINITION OF EQUIVALENT CONDUCTIVITY

In order to solve this problem, a particular definition of the equivalent conductivity can be used in order to link the losses in both cases, even with completely different EC distributions. This approach is possible because the global EC losses in a homogeneous material are directly proportional to the constitutive conductivity.

## A. EC losses in the cell

In the present case, the elementary cell is made of two materials with different conductivities. Since EC only appear in the inclusion, the determination of the losses in the material reduces to the calculation of the losses in the square-shaped grain of material 2:

$$U_{cell} = \left(\frac{L_2}{L_1}\right)^2 \cdot \frac{\langle \mathbf{E}^T \overline{\sigma} \mathbf{E} \rangle_2}{2f} = v_2 \cdot \frac{\langle \mathbf{E}^T \overline{\sigma} \mathbf{E} \rangle_2}{2f}$$
(2)

where  $v_2$  is the volume fraction of material 2. The operator  $\langle \cdot \rangle_2$  denotes a volume average over the grain. The electric field in a homogeneous square resulting from the application of a perpendicular magnetic field can be derived analytically [10]. With the assumption of low frequency ( $\omega \epsilon_2 \ll \sigma_2$ ), the expression can be simplified into:

$$U_{cell} = v_2 \frac{9\pi^2}{128} f \sigma_2 B_2^2 L_2^2 \tag{3}$$

where  $B_2$  is the magnitude of the magnetic induction in the grain  $(B_2 = (\mu_2/\tilde{\mu}) \cdot B$  with *B* the average magnetic induction and  $\tilde{\mu}$  the Wiener effective permeability).

#### B. EC losses in the homogeneous structure

Whatever the 2D structure, the losses in a homogeneous material (with conductivity  $\sigma$ ) subjected to a perpendicular magnetic flux always has a proportion relation:

$$U_{struct} \propto f \cdot \sigma \cdot B^2 \tag{4}$$

Here are some examples for simple shapes with low frequency assumption:

$$U_{struct} = \begin{cases} \frac{9\pi^2}{128} f\sigma B^2 L^2 & \text{for a square (side L)} \\ \frac{\pi^2}{6} f\sigma B^2 L^2 & \text{for a plate (widthL) (5)} \\ \frac{\pi^2}{4} f\sigma B^2 R^2 & \text{for a disk (radius R)} \end{cases}$$

## C. Equivalent conductivity

For a real square-shape structure of SMC, the equivalent conductivity  $\hat{\sigma}$  is obtained by equalizing Eq. (3) and Eq. (5). This leads to:

$$\hat{\sigma} = v_2 \sigma_2 \left(\frac{L_2 \mu_2}{L\tilde{\mu}}\right)^2 \tag{6}$$

For a given size L, it can be noticed that the smaller the grain size, the smaller the equivalent conductivity, meaning that small grain sizes give low losses, which is consistent with the SMC properties.

For more complex geometries, a FEM computation with homogeneous conductivity is necessary to identify the proportionality coefficient for  $U_{struct}$ . The equivalent conductivity can then be defined. The methodology will be detailed in the full paper on more complex configurations for both the periodic cell and the geometry.

## IV. CONCLUSION

A homogenization strategy is proposed to define the equivalent conductivity for SMC based on an estimate of EC losses. Even if the EC distribution is very different between homogeneous and heterogeneous materials, an estimate of average EC losses can be obtained with this approach.

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